Math 2050, HW 3

(1) If $x_1 = 1$, $x_2 = 2$ and $x_{n+2} = \frac{1}{3}x_{n+1} + \frac{2}{3}x_n$, show that $\{x_n\}_{n=1}^{\infty}$ is convergent and find its limit.

(2) Prove or disprove the following: Suppose $\sum x_n$ is a convergent series with $x_n > 0$ for all n.
   (a) $\sum x_n^2$ is convergent.
   (b) Is $\sum \sqrt{x_n}$ is convergent.

(3) If $f : \mathbb{R} \to \mathbb{R}$ is a function given by $f(x) = x$ for $x \in \mathbb{Q}$ and $f(x) = 0$ for $x \notin \mathbb{Q}$, then f is continuous at $x = 0$.

(4) (a) Show that $\lim_{x\to 3} \frac{2x+3}{4x-9} = 3$.
   (b) Determine if $\lim_{x\to 0^+} \sin(x^{-1})$ exists.
   (c) Determine if $\lim_{x\to 0} x \sin(x^{-2})$ exists.